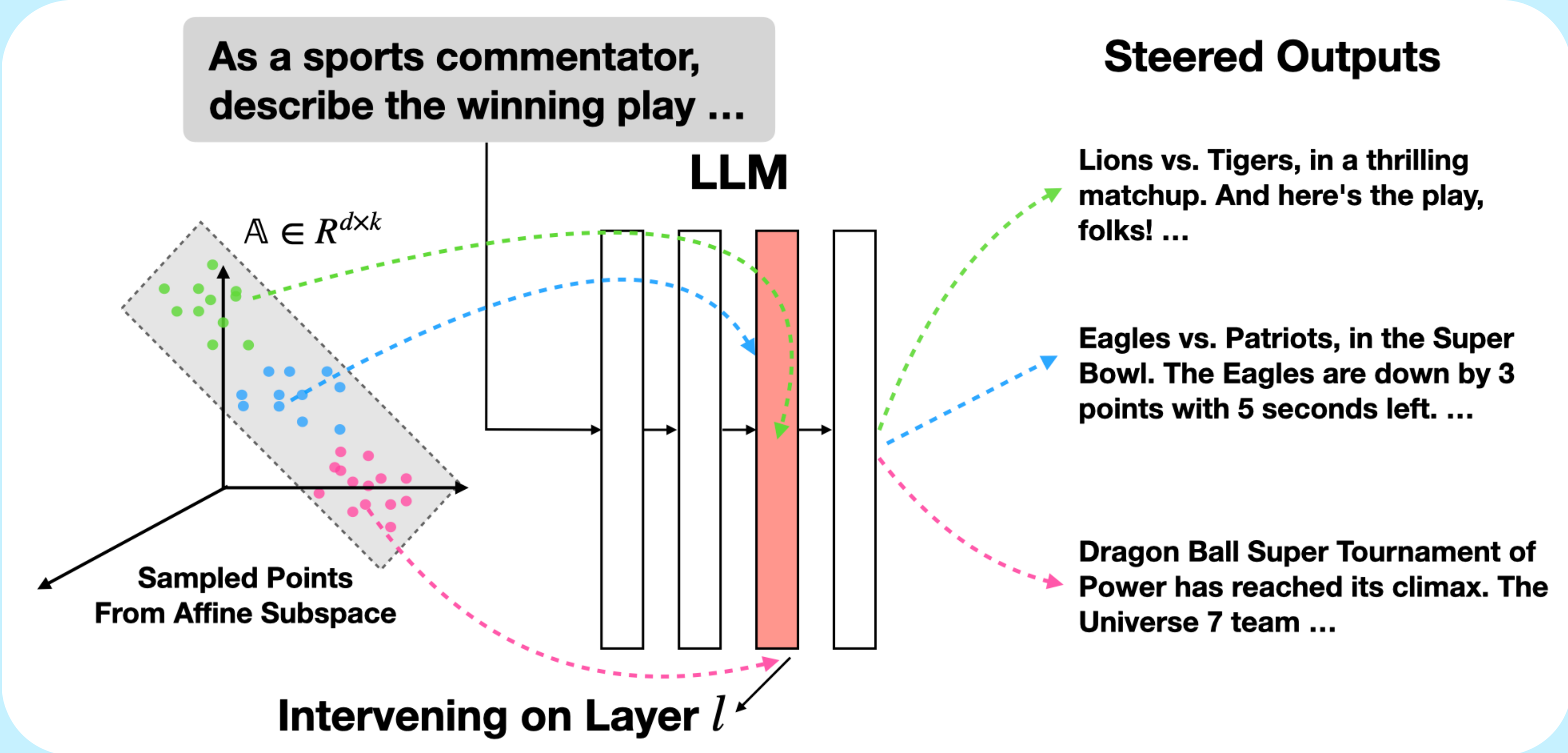


# Localizing and Evaluating Multinomial Concepts with Affine Subspaces

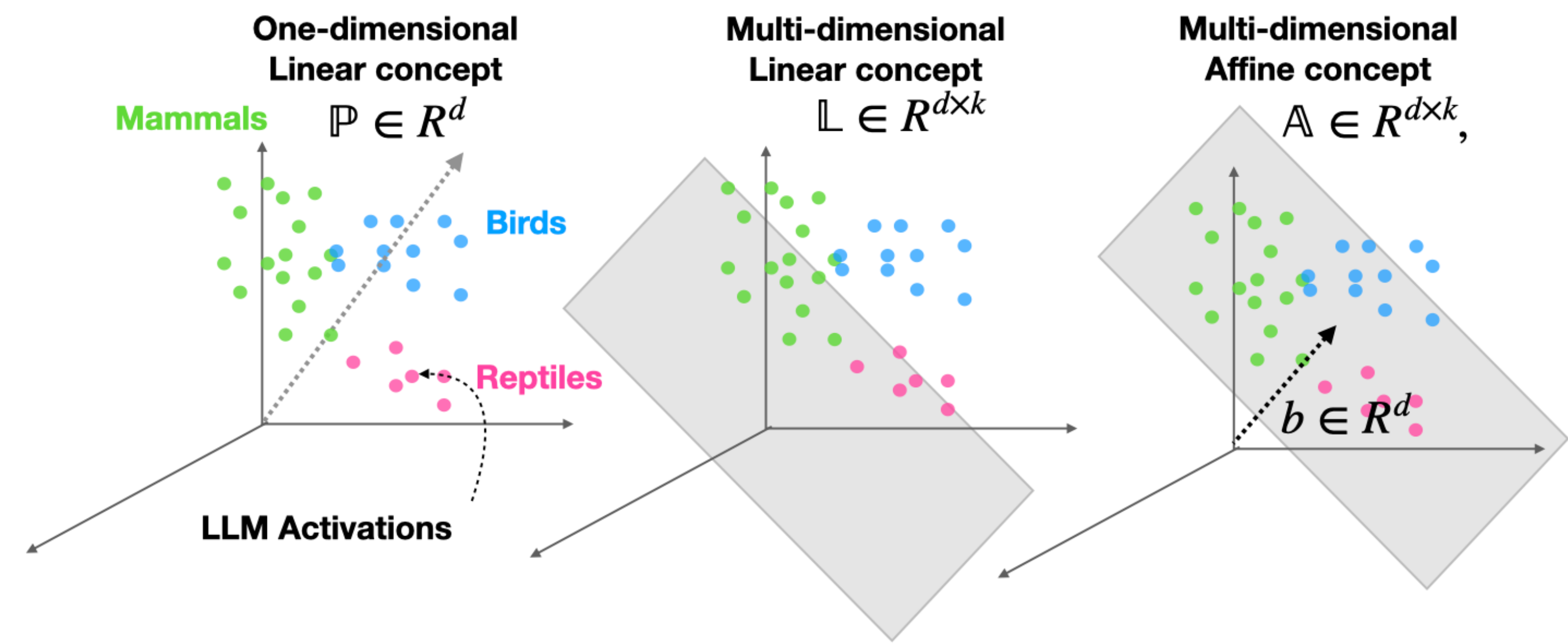
Divya Appapogu<sup>1</sup>, Freya Behrens<sup>1</sup>, Yonatan Belinkov<sup>2</sup>, Aaron Mueller<sup>1</sup>

## TL;DR

Multidimensional affine concept representations enable richer steering than one-dimensional representations: sampling within the concept space covers the training distribution while generating coherent, diverse novel instances.

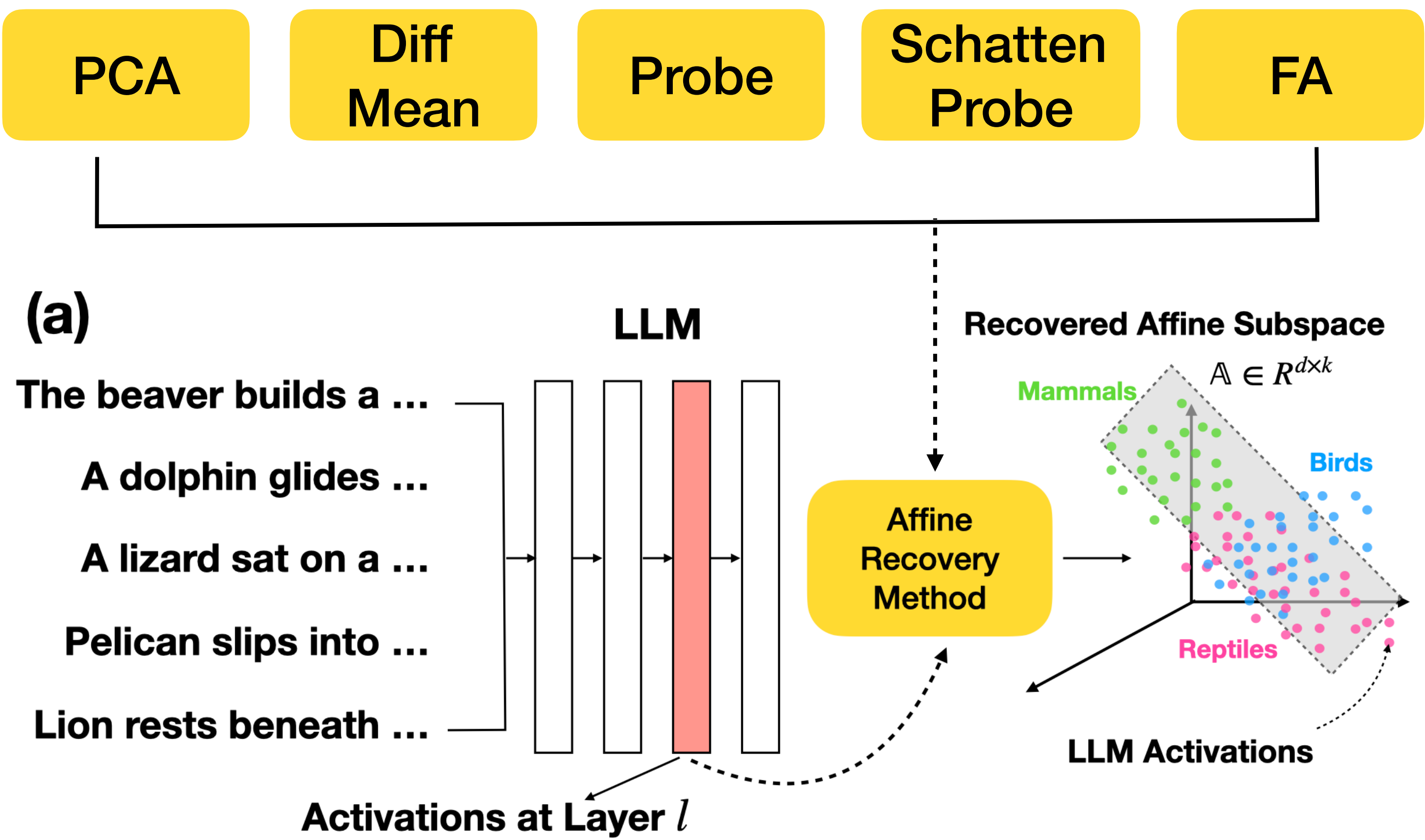


## Motivation



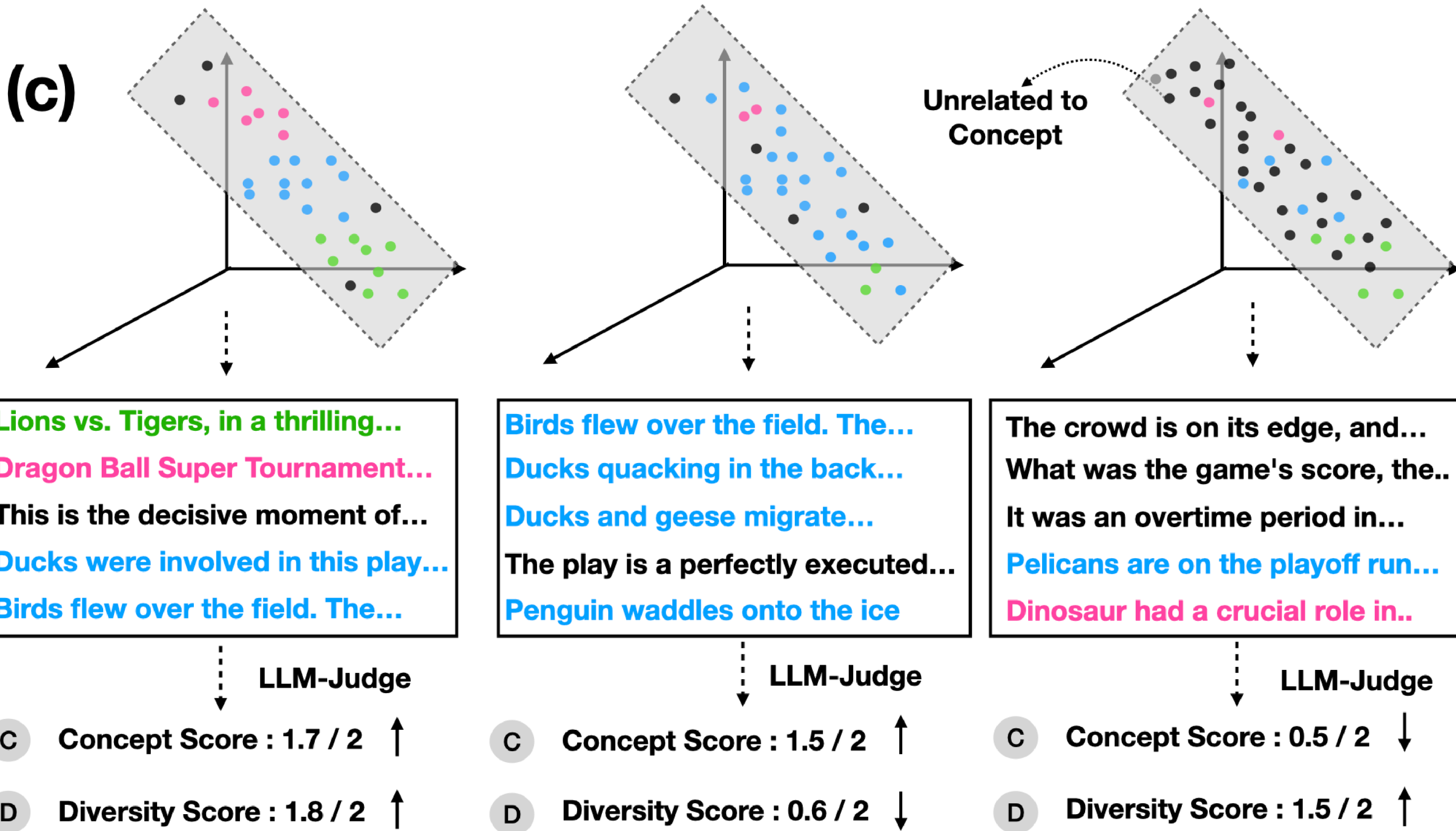
The linearity of a concept is an elegant but strong assumption. What do we gain in a multidimensional setting?

## Methods to Recover Affine Subspace



## Concept and Diversity Measures

We use LLM Judge to evaluate steered outputs for concept and diversity scores.



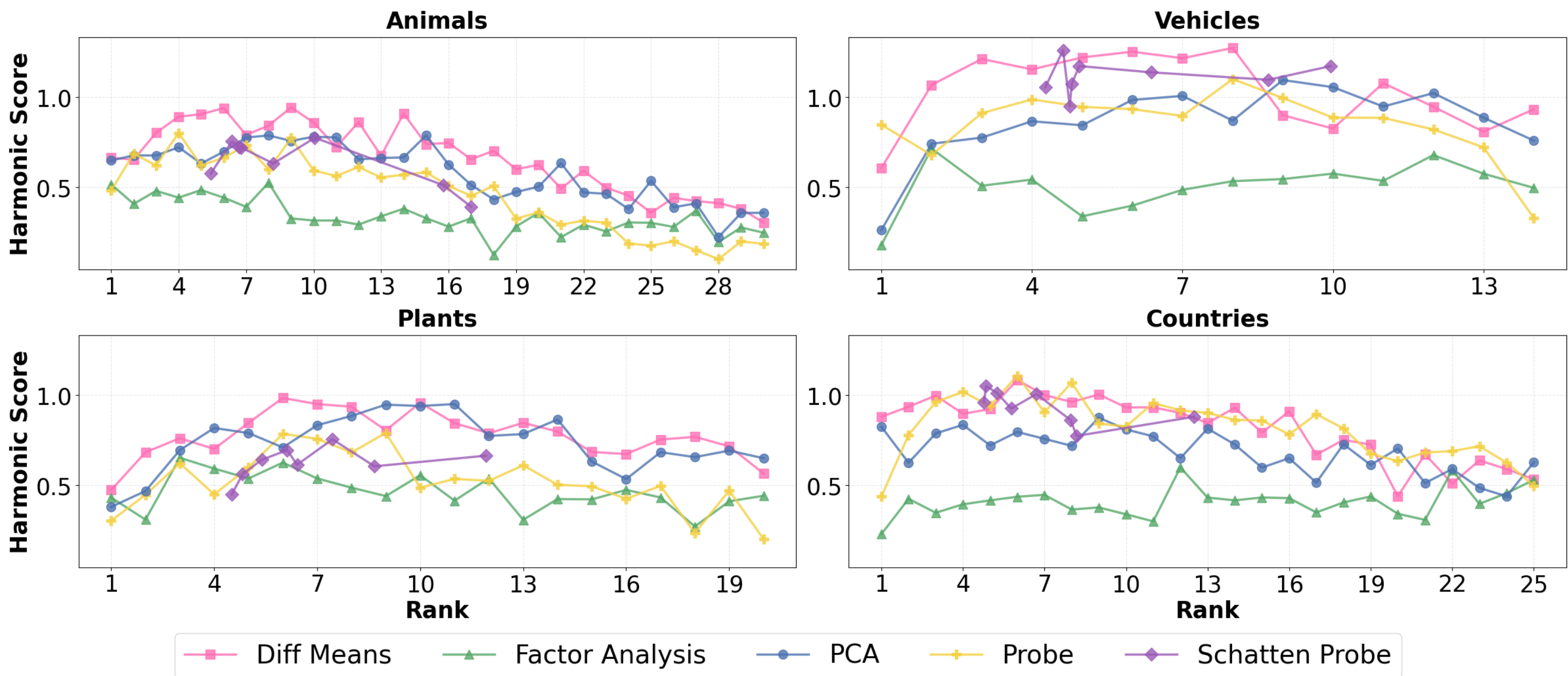
## Experiments

For Llama, Factor Analysis and schatten probe perform best. For Gemma, DiffMean and both probes perform well. All methods outperform prompting.

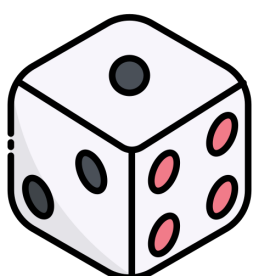
| Model        | Method         | ANIMALS      | VEHICLES     | PLANTS       | COUNTRIES    |
|--------------|----------------|--------------|--------------|--------------|--------------|
| Llama-3.1-8B | Prompting      | 0.630        | 0.366        | 0.463        | 0.958        |
|              | DiffMean       | 0.913        | 1.153        | 0.790        | 1.432        |
|              | FA             | 1.023        | <b>1.259</b> | <b>0.852</b> | <b>1.520</b> |
|              | PCA            | 0.936        | 1.048        | 0.808        | 1.380        |
|              | Probe          | 1.028        | 0.955        | 0.725        | 1.369        |
|              | Schatten Probe | <b>1.076</b> | 1.114        | 0.768        | 1.424        |
| Gemma-2-2B   | Prompting      | 0.480        | 0.006        | 0.192        | 0.642        |
|              | DiffMean       | <b>0.944</b> | <b>1.273</b> | <b>0.984</b> | 1.084        |
|              | FA             | 0.527        | 0.716        | 0.653        | 0.599        |
|              | PCA            | 0.789        | 1.095        | 0.950        | 0.875        |
|              | Probe          | 0.802        | 1.101        | 0.791        | <b>1.108</b> |
|              | Schatten Probe | 0.776        | 1.261        | 0.755        | 1.050        |

| VEHICLES       |              |              |              |
|----------------|--------------|--------------|--------------|
| Method         | Concept      | Diversity    | Harmonic     |
| Prompting      | 0.040        | 0.200        | 0.006        |
| DiffMean       | <b>0.973</b> | <b>1.840</b> | <b>1.273</b> |
| FA             | 0.467        | 1.535        | 0.716        |
| PCA            | 0.800        | 1.737        | 1.095        |
| Probe          | 0.800        | 1.763        | 1.101        |
| Schatten Probe | <b>0.973</b> | 1.791        | 1.261        |

### Rank-wise Analysis for Gemma-2-2B



## Takeaways



Sampling can be used as evaluation metric for concepts without known ground-truth geometry the common case in LLMs.



Diverse sampling within a learned subspace enables targeted, controllable interventions on LLM outputs.